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13. ABSTRACT *(Maximum 200 words)*

This research project has addressed critical issues concerned with the mathematical modeling of phase transitions in solids. Detailed microstructural analysis has enabled us to clarify the role of stress waves in the propagation of phase boundaries. The potential for phase boundary movement to damp out stress waves has been demonstrated for certain classes of material response. Nucleation of new phase domains by highly energetic processes has been successfully modeled and new analytical procedures have been developed for the predictive response of phase transforming media during high energy impact. Conversion of mechanical energy to thermal energy has been studied by means of an extended theory which incorporates temperature effects. The role of these dynamical events on the response of devices at the engineering level points to the utility of a mathematical description capable of capturing cumulative microstructural effects. To this end we have also developed mathematical protocols capable of tracking the evolution of thermomechanical austenite/martensite phase variants due to generalized conditions of loading and heating. The associated mathematical model is capable of capturing superelastic response and two-way shape memory in thermoelastic materials. Ohmic heating/convective cooling is a likely activation mechanism for smart devices utilizing these materials, and our numerical simulations have successfully replicated these processes including a smooth transition from isothermal to adiabatic response as the loading rate is increased in a heat convective environment. The effect of incorporating these highly nonlinear materials in composite layered arrangements and thin film structures has also been examined.

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Phase transitions, nonlinear partial differential equations, wave propagation, loss of ellipticity and change of type, hysteresis, dissipation, energy minimization, structural optimization, composite materials, shape memory effect, smart materials.

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Final Report: Rapidly Activated Dynamic Phase Transitions In Nonlinear Solids

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Abstract: This research project has addressed critical issues concerned with the mathematical modeling of phase transitions in solids. Detailed microstructural analysis has enabled us to clarify the role of stress waves in the propagation of phase boundaries. In particular, the potential for phase boundary movement to damp out stress waves has been demonstrated for certain classes of material response. Nucleation of new phase domains by highly energetic processes has been successfully modeled and new analytical procedures have been developed for the predictive response of phase transforming media during high energy impact. Conversion of mechanical energy to thermal energy has been studied by means of an extended theory which incorporates temperature effects. The role of these dynamical events on the response of processes and devices at the engineering level points to the utility of a higher order mathematical description capable of capturing cumulative microstructural effects. To this end we have also developed mathematical protocols capable of tracking the evolution of thermomechanical austenite/martensite phase variants due to generalized conditions of loading and heating. The associated mathematical model is capable of capturing superelastic response and two-way shape memory in thermoelastic materials. Ohmic heating/convective cooling is a likely activation mechanism for smart devices utilizing these materials, and our numerical simulations have successfully replicated these processes including a smooth transition from isothermal to adiabatic response as the loading rate is increased in a heat convective environment. The effect of incorporating these highly nonlinear materials in composite layered arrangements and thin film structures has also been examined. This research project has, to date, generated eleven archival papers and conference proceedings, contributed to eighteen technical presentations, and supported two Ph.D. graduate students.

1. Introduction

The purpose of this research project, as stated in the original research proposal, was to address *"critical issues concerned with the mathematical modeling of phase transitions in solids. The specific phase transitions of interest are those which are rapidly activated by either mechanical or thermal means. The influence of stress waves will then exert a fundamental role on the triggering and subsequent propagation of phase boundaries. At present, the ability to predict the dynamical response of phase transforming materials is insufficiently precise for confidence in understanding how these materials will react under extreme conditions or perform in novel design settings. This research is designed to develop analytical methods for addressing these issues."* The original research proposal then proceeded to describe in more detail the specific problems to be examined and the methods proposed for their treatment.

We are happy to report that these goals have been met. Substantial progress has been achieved on all of the problem areas proposed for original study. This is evidenced by the numerous publications and technical presentations which have resulted from this research. Two graduate students, Jack Lin and Ralph Worthington, have benefitted from intense involvement in this research. It is anticipated that Jack will receive his Ph.D. in May 1993 and that Ralph will receive his Ph.D. sometime in 1994. In addition, our accomplishments have suggested new avenues of inquiry, some of which we have also begun to pursue. In particular, we have initiated collaboration with an industrial research partner which has made possible a much faster degree of technical transfer than that which would be possible by the traditional publication & presentation process (this industrial collaboration has also resulted, to date, in two soon-to-appear publications). The support of ARO has also permitted us to obtain a certain amount of leveraged research funding from the State of Michigan Research Excellence Fund ($24,991, 10/89-9/90), ($24,000, 10/90-9/91), ($27,000, 10/91-9/92), ($31,911, 10/92-6/93) to extend some of the issues addressed in our ARO study to the domain of composite material structures.

The support provided by this research grant has been instrumental to the success of the principal investigator's academic career during the past three years. During this time 11 papers supported by ARO have either appeared, or have been accepted for publication, in the journals of record for mechanics and applied mathematics (including the *Archive for Rational Mechanics and Analysis*, the *Journal of Elasticity*, and the *International Journal of Engineering Science*), and additional papers are in various stages of preparation. Eighteen presentations have been delivered at external conferences and other universities. The principal investigator was promoted by his institution from Assistant Professor to Associate Professor with tenure, and was also awarded the College of Engineering Withrow Teaching Excellence Award in 1991, the inaugural year for this award. In 1992, Ph.D. graduate student Jack Lin was recognized by the College of Engineering as the outstanding graduate student in the Department of Materials Science and Mechanics.

During the period of time encompassed by this research (8-1-89 to 12-15-92) we have, of course, continued to monitor the research advances obtained by other investigators in this area (some of whom are also supported by ARO), and our research has benefitted from this scholarly interaction. Now that the success of our mathematical modeling procedures has been established, we are beginning to seek avenues for the additional application of our results. To this end we have begun collaboration with materials scientists whose specialities are in materials fabrication and processing. In particular, as of this writing, research proposals are pending to hopefully enable us to pursue such extensions (a research proposal entitled *Mathematical Modeling and Characterization of Self-Biasing Thermoelastic Thin Film Microlaminates on High Temperature Polymeric Substrates*, co-authored with materials scientist D.S. Grummon, has been submitted to the ARO's URI-RIP 93 program (control number SM-13548)).

In the next two sections of this final report we list the publications that have already resulted from this research project, as well as the external presentations delivered during the granting period. Then in the next five sections we summarize in more detail our various technical achievements. In this description we underscore not only how we have fulfilled the goals of the originally proposed

project, but also how new results, both by us and others, have been successfully incorporated into our research program. We also touch upon some of the new avenues of opportunity opened by our endeavors.

2. Publications Acknowledging ARO Support

The following eleven papers all acknowledge ARO support and have either appeared or been accepted for publication. These papers have been submitted to the ARO library as per the grant specifications (except for the Transactions of the Army Conferences which originated from ARO publication sources). In addition, other manuscripts are presently in various stages of preparation which will also acknowledge ARO support.

Ivshin, Y. and T. J. Pence (1992a). A constitutive model for hysteretic phase transition behavior. (accepted for publication in *Int. J. Eng. Sci.*).

Ivshin, Y. and T. J. Pence(1992b). A simple mathematical model of two-way memory effect. (to appear in *Proceedings of the 1992 Int. Conf. on Martensitic Transformations* (ICOMAT-92)).

Lin, J. and T. J. Pence (1991). Energy dissipation in an elastic material containing a mobile phase boundary subjected to concurrent dynamic pulses. Transactions of the Ninth Army Conference on Applied Mathematics and Computing, F. Dressel ed., 437-450.

Lin, J. and T. J. Pence (1992a). On the dissipation due to wave ringing in non-elliptic elastic materials. (accepted for publication in *J. Nonlinear Sci.*).

Lin, J. and T. J. Pence (1992b), Kinetically driven elastic phase boundary motion activated by concurrent dynamic pulses, to appear in Transactions of the Tenth Army Conference on Applied Mathematics and Computing, F. Dressel ed.

Pence, T.J. (1990a). Phase transitions and maximally dissipative dynamic solutions in the Riemann problem for impact. Transactions of the Eighth Army Conference on Applied Mathematics and Computing, F. Dressel ed. 817-832.

Pence, T.J. (1990b). An inverse Riemann problem for impact involving phase transitions. Transactions of the Eighth Army Conference on Applied Mathematics and Computing, F. Dressel ed. 833-846.

Pence, T.J. (1991b). On the encounter of an acoustic shear pulse with a phase boundary in an elastic material: energy and dissipation. *J. Elasticity* **26**, 95-146.

Pence, T.J. (1992a). On the mechanical dissipation of solutions to the Riemann problem for impact involving a two-phase elastic material. *Arch. Ration. Mech. & Anal.* **117**, 1-52.

Pence, T.J. (1992b). The dissipation topography associated with solutions to a Riemann problem involving elastic materials undergoing phase transitions. to appear in *Shock Induced Transitions and Phase Structures in General Media*, R. Fosdick, E. Dunn & M. Slemrod eds., IMA volume series, Springer-Verlag.

Song, J. and T. J. Pence (1992), On the design of three-ply nonlinearly elastic composite plates with optimal buckling performance, *Structural Optimization* **5** (1992) 45-54

3. External Presentations During the Grant Period

Eighteen presentations were given at technical conferences, or delivered during invited seminars at other universities:

XXI Midwestern Mechanics conference, August 13-16, 1989, Houghton, MI.

SES 26^{th} Annual Meeting, September 18-20, 1989, Ann Arbor, MI.

Invited seminar, Department of Mechanical Engineering, The Johns Hopkins University, Oct. 12, 1989.

Invited seminar, Department of Applied Mathematics, University of Virginia-Charlottesville, Oct. 13, 1989.

Society for Natural Philosophy 34^{th} Meeting, April 6-8, 1990, Lincoln, NB.

Eleventh National Congress of Applied Mechanics, (with J. Song), May 21-25, 1990, Tucson, AZ.

Eleventh National Congress of Applied Mechanics, (with Y. Ivshin), May 21-25, 1990, Tucson, AZ.

American Society of Composites 5^{th} Technical Conference, (with J. Song), June 12-14, 1990, East Lansing, MI.

Eighth Army Conference on Applied Mathematics and Computing, June 19-22, 1990, Ithaca, NY.

IMA workshop: Shock Induced Transitions and Phase Structures in General Media, Oct. 15-19, 1990, Minneapolis.

Invited lecture, Dept. of Aerospace Engineering and Applied Mechanics, University of Minnesota, Oct. 30, 1990.

One hundred eleventh Winter Annual Meeting of ASME, Nov. 25-30, 1990, Dallas, TX.

1991 Mathematica Conference, (with Y. Ivshin), January 12-15, 1991, San Francisco, CA.

Invited seminar, Depts. of Civil Engineering and Mechanical Engineering, Northwestern University, May 17, 1991.

Ninth Army Conference on Applied Mathematics and Computing, June 18-21, 1991, Minneapolis, MN.

SES 28^{th} Annual Meeting, November 6-8, 1991, Gainesville, FL.

Tenth Army Conference on Applied Mathematics and Computing (with J. Lin), June 16-19, 1992, West Point, NY.

Int. Conference on Martensitic Transformations, (with Y. Ivshin), July 20-24, 1992, Monterey, CA.

4. Phase Boundary Propagation in the Presence of Stress Waves

As background, phase transformations in solids can be modeled in a purely equilibrium setting within the finite deformation theory of elasticity provided that the elastic potential energy density of the material admits a multiple well structure. This allows the governing equations to lose ellipticity, consequently the solutions of boundary value problems may then contain internal surfaces of strain discontinuity that separate distinct phase regions. Phase boundaries of this type, and the attendant configurations involving co-existing solid phases in equilibrium, have been studied by numerous authors, including Knowles and Sternberg (1978), James (1979, 1981, 1983, 1986), Kikuchi and Triantafyllidis (1982), Fosdick and Macsithigh (1983), Gurtin (1983), Abeyaratne and Knowles (1987a, b, 1988a, b), Silling (1989) and Jiang (1991). A consistent feature of these studies is that boundary value problems which would be well-posed in the conventional "elliptic theory" (*viz.* Ball 1977) often become underposed in the "non-elliptic theory": namely a massive loss of uniqueness prevails if only the usual constitutive and mechanical equations of equilibrium are required to hold.

These different equilibrium configurations will typically each store a different amount of mechanical energy. In certain cases, configurations that are global energy minimizers may be identified, although it also may be the case that more than one configuration provides a global energy minimum state (Ericksen 1975). In other cases, energy minimal equilibrium configurations might not exist, in the sense that a lower bound energy level is approachable but not attainable. This may explain the phenomena of "frustration" in magnetic materials (James and Kinderlehrer 1990). In addition, when the governing energy densities explicitly account for the crystallographic symmetries of the competing phases, the microstructural "quest" for an unattainable minimum energy state

might then be responsible for fine variations in crystallographic domain structure (Ball and James 1987, 1990). Energy based methods for determining optimal microstructural types and arrangements of phases, voids and other defects are presently an active area of study (e.g. Collins, Kinderlehrer and Luskin 1991; Collins and Luskin 1991; James and Spector 1990; Kohn 1989, 1991; Bhattacharya 1991; Fosdick and Zhang 1992). The presence of hysteresis and generalized "memory" effects in the quasi-static evolution of equilibrium states suggests that energy minimization might not be the sole governing agent for the determination of equilibrium. It has been suggested (Abeyaratne and Knowles 1988a) that kinetic balance relations operating at phase boundary interfaces may then control the evolution of equilibrium states.

If the dynamics of these systems is purely elastic, then the governing field equations and discontinuity conditions involve only the addition of inertial terms. Phase boundary motion is possible in this setting and the underposedness problem remains (James 1980; Shearer 1982, 1986, 1988; Slemrod 1983, 1989; Hattori 1986a,b; Pence 1986, 1987; Truskinovsky 1987, 1990). In fact if one considers a body in equilibrium which is then dynamically disturbed, there are now two possible sources of nonuniqueness: nonuniqueness in initial conditions stemming from those issues that are resident in the equilibrium theory as mentioned above, and an additional purely dynamical nonuniqueness due to the fact that the dynamical motion from a *fixed* set of initial conditions may also admit multiple solutions for $t > 0$.

Turning now to dynamical nonuniqueness, a *moving* phase boundary in one space dimension is in general either a point source or a point sink for mechanical energy. Standard interpretations of the second law of thermodynamics then require that only sink-like, or dissipative, motion is admissable (Dafermos 1973; Abeyaratne and Knowles 1990a; Gurtin 1991). This restriction is typically not sufficient to wholly resolve issues of dynamical uniqueness, and additional criteria have been proposed for this purpose including: requirements of maximizing energy dissipation (Dafermos 1973, 1989; James 1980; Hattori 1986a), and the application dynamical kinetic relations governing phase boundary mobility (Truskinovsky 1987; Gurtin and Struthers 1990; Abeyaratne and Knowles 1991).

Dynamical nonuniqueness is not at issue in the event that the equations of motion involve not only the inertial terms of the purely elastic theory, but also an additional viscosity term (Slemrod 1983; Pego 1987; Ball *et al.* 1991). The equilibrium theory is not affected and the regularization provided by the higher order derivative delivers a unique solution to problems that begin from a fixed set of initial conditions. In addition, viscous dissipation takes place throughout the field. One might thus expect that these dynamical solutions would then have two dissipative mechanisms: viscous dissipation throughout the field and localized energy sinks provided by moving phase boundaries. In fact there is only the viscous field dissipation, as the viscous regularization neither permits the motion of initial phase boundaries nor allows for creation of new phase boundaries (Proposition 3.5 and Theorem 3.6 of Pego 1987; Theorem 4.10 of Ball *et al.* 1991). Modeling of other physical effects, such as capillarity (Hagan and Slemrod 1983) or a short thermal memory (Niezgodka and Sprekels 1988; Niezgodka, Zheng and Sprekels 1988) can give rise to additional higher order derivative terms in the governing equations; the solutions of these equations may then be sensitive to various scalings of these terms even when their individual contributions are vanishingly small (Abeyaratne and Knowles 1990b).

This drastic suppression of phase boundary motion in models which utilize viscosity terms in the governing equations of motion is at odds with the observed character of diffusionless phase trans-

formations. The austenite-martensite phase boundary, for example, is a sharp interface which is found on occasion to travel at speeds approaching the shear wave speed of the material (Bunshak and Mehl 1952; Grujicic, Olson and Owen 1985). This supports the consideration of dynamical models for phase transitions that involve elastic dynamics supplemented by appropriate local criteria for phase boundary mobility. In Pence (1991a) we utilized this framework to investigate the role of shear waves in driving a phase boundary through a two-phase material. Specifically, we considered the interaction of a finite amplitude acoustic shear pulse with a pre-existing phase boundary in a material that was in equilibrium prior to the introduction of the pulse. In general, from this encounter there eventually emerges both a reflected pulse and a transmitted pulse. The dynamical fields associated with the different possibilities for phase boundary motion were completely parametrized in terms of the phase boundary speed. For certain ranges of initiating shear pulse amplitude, a completely reflecting solution was found, while for an in general different range of shear pulse amplitudes a completely transmitting solution was found. The properties of these different solutions suggest that the notion of phase boundary impedance may be useful in determining criteria for phase boundary mobility.

Subsequently (Pence 1991b) these results were examined from the perspective of energy and dissipation. It was shown that there exists at most two motions which involve no dissipation (corresponding to conservation of mechanical energy), that there exists a third motion which maximizes the mechanical energy dissipation rate, and that there exists yet a fourth motion which maximizes the total energy loss during the encounter (the distinction between the third and fourth motions is due to the fact that certain encounters with lower dissipation rate have longer duration). This raises the issue of whether or not the disturbance applied to an energetically stable equilibrium state provided by such a finite amplitude shear pulse can, through subsequent shear wave reverberations, *completely* damp itself out, thus allowing the material to assume a new energetically stable minimum state consistent with the external load on the system after the pulse has been generated. Nondissipative motion does not allow for this possibility. However it was established (Pence 1991b) that this possibility is not excluded for either the third (maximally dissipative) motion, or the fourth (maximal loss) motion. In fact, the optimal energy loss for a single maximally dissipative encounter is 70.5% of that needed to attain the new ground state, and this occurs within materials for which the transformation strain is 96.7% of the initiating pulse amplitude. These results put rigorous bounds on the possibilities for energy absorption via a single (and hence rapid) phase transformation event. The energetic behavior of a phase boundary that is subjected to concurrent dynamic pulses, one from each side, for a maximally dissipative motion is examined in Lin and Pence (1991), where the total energy loss is contrasted to that which would occur if the two pulses were not concurrent. It is found that the concurrent pulse encounter suffers the greater energy loss in the event that both incoming pulses are of the same sign (with respect to strain), whereas the concurrent pulse encounter suffers the lesser energy loss in the event that the incoming pulses are of opposite sign.

The ultimate damping capacity for these dynamical processes is dependent upon the large-time dissipative behavior of the ensuing shear wave reverberations. The essential obstacle to an explicit analysis of this problem is that each pulse-phase boundary encounter spawns both a reflected pulse and a transmitted pulse, thus giving rise to distinct pulse generations, involving a geometric increase in the number of pulses which must be treated in each generation. Faced with this difficulty, we developed (Lin and Pence, 1992a) an analytical framework which abandons the notion of tracking the space-time trajectory of each individual pulse, and instead monitors the ongoing change in

the stored dynamical field energy as a function of initiating pulse magnitude and reverberation generation number. Numerical simulations utilizing this scheme have enabled us to examine the question raised above regarding the possibility of completely damping out an initial dynamical perturbation. We establish that both the maximally dissipative motion, and the maximal energy loss motion, dissipate exactly the amount of energy needed for the dynamical system to settle into the new energy minimal equilibrium ground state associated with the final value of load. The numerical simulations indicate how the energy loss proceeds by reverberation generation number, and demonstrates that the energy remaining at each generation finely distributes itself over a geometrically increasing number of ever smaller pulses. These results are established numerically for arbitrary sized initiating pulse; we were also able to obtain the same results analytically for the special case of a vanishingly small dynamical perturbation. This latter problem is rendered meaningful by scaling the initiating pulse magnitude with respect to the transformation strain before extracting the limit. In fact, it was found that, in this limit, each generation dissipates exactly one half of the excess energy remaining within the system. This intriguing result can be contrasted, for example, with the optimal result for a maximally dissipative motion in which 70.5% of the energy is dissipated within the first generation. These results are apparently the first of any kind dealing with the large time dissipative behavior of systems governed by an elastic dynamics supplemented with a *local* criterion for phase boundary mobility. As such they form a useful counterpart to the results, given recently in Ball *et al.* (1991), for the large time dissipative behavior of systems governed by field equations with an explicit viscous dissipation. It is to be noted that the dissipative mechanism is field resident in Ball *et al.* (1991) and hence *global* in action, whereas in our case the dissipative mechanism is inherently local.

The problems discussed in Lin and Pence (1991, 1992a) have focused on the maximum dissipation rate criterion for determining the mobility of a propagating phase boundary. This has clarified the ultimate damping capacity of these materials and, in particular, indicates how dynamical phase transition processes can settle down into equilibrium states governed by considerations based upon energy minimization. Phase boundary mobility can also be regarded as governed by a deterministic kinetic relation between the phase boundary velocity and the local interface driving traction (since the latter is the dissipation rate divided by the phase boundary velocity, this is also equivalent to a relation between the phase boundary velocity and the dissipation rate). Noncontinuum theories (atomistic and statistical mechanical) provide guidance in the formulation of such relations. The proper thermodynamical framework for the incorporation of a kinetic relation has been clarified by Gurtin and Struthers (1990) and the stability of kinetically driven phase boundary motion has been examined by Fried (1991a,b). Abeyaratne and Knowles have examined the role of a kinetic relation in determining phase boundary motion in both the quasi-static (1988a) and the fully dynamic (1991) setting.

To the extent that the maximum dissipation rate criteria provides a relation between dissipation and phase boundary velocity (Pence 1991b) it can be regarded as a kinetic relation, although it is our belief that the maximum dissipation rate criterion is more properly viewed as providing the limiting behavior for dissipative behavior from among all possible kinetic relations. In view of our present understanding of this limiting behavior as given in Pence (1991a,b) and Lin and Pence (1992a), and because of the greater generality of the kinetic relation point of view, we have begun to investigate the extent to which this limiting dissipative behavior is attainable from kinetically driven phase boundary propagation. To this end, in Lin and Pence (1992b) we have reexamined the concurrent pulse problem for the case in which phase boundary mobility is governed by a kinetic relation instead of a maximum dissipation condition. Specifically, we have considered a linear rela-

tion $\dot{s} = \kappa f$ where $\dot{s}$ and f are the phase boundary velocity and the phase boundary driving traction, and $\kappa > 0$ is a linear coupling constant. The cases $\kappa \to \infty$ and $\kappa \to 0$ then correspond to an energy conserving motion and a rigidly immobile phase boundary respectively. As in Lin and Pence (1991) the question of interest is to determine whether the greater energy loss occurs when the pulses act concurrently or separately. If the incoming pulses are of opposite sign, we again find that the greater energy loss occurs when the pulses act separately. However if the incoming pulses are of the same sign, then the greater energy loss is experienced by the concurrent interaction *only* if each of the incoming pulses is sufficiently small. If the incoming pulses are of the same sign, and one or the other is sufficiently large, then the greater energy loss takes place when the pulses act separately. These results are established by a combination analytical/numerical analysis which allows us to calculate the threshold values on pulse size which lead to these different behaviors. Work is now continuing in the analysis of the large-time behavior of these kinetically driven ringing processes and it is clear that at least one more major archival paper will result from our progress to date.

This section summarizes results that are reported in the following four publications:

Lin, J. and T. J. Pence (1991). Energy dissipation in an elastic material containing a mobile phase boundary subjected to concurrent dynamic pulses. Transactions of the Ninth Army Conference on Applied Mathematics and Computing, F. Dressel ed., 437-450.

Lin, J. and T. J. Pence (1992a). On the dissipation due to wave ringing in non-elliptic elastic materials. (accepted for publication in *J. Nonlinear Sci.*).

Lin, J. and T. J. Pence (1992b), Kinetically driven elastic phase boundary motion activated by concurrent dynamic pulses, to appear in Transactions of the Tenth Army Conference on Applied Mathematics and Computing, F. Dressel ed.

Pence, T.J. (1991b). On the encounter of an acoustic shear pulse with a phase boundary in an elastic material: energy and dissipation. *J. Elasticity* **26**, 95-146.

In addition, another manuscript is in preparation which we intend to submit to the *Journal of the Mechanics and Physics of Solids* in late Spring 1993.

5. Phase Boundary Creation Due to Impact

In our work that we have outlined in Section 4, phase transformations were due to the motion of a pre-existing phase boundary. Equally important is the generation of new phase boundaries. To clarify this aspect of phase transformation we have examined impact induced phase transitions (Davison and Grahm 1979; Asay and Kerley 1987). Immediate attention was restricted to models for isothermal phase transitions. The work of Slemrod (1983, 1984) and Hattori (1986a,b) indicates how these isothermal models serve to clarify the overall analytical issues before treating more generalized models (to be discussed in Section 6). Even in this preliminary setting, major challenges must be confronted due to the possibilities for loss of hyperbolicity in the governing partial differential equations (Liu 1975; Schaeffer and Shearer 1987; Shearer, Schaeffer, Marchesin and Paes-Leme 1987; Issacson, Marchesin, Plohr and Temple 1988). In view of this challenge, it is common to adopt restrictions on the size of the variation of the initial data (e.g. Hattori 1986a). However, motivated by situations in which the impact velocity may be large, it was our desire to obtain a methodology that does not hinge upon any such requirement. We achieved this goal for the fundamental Riemann problem by focusing on separate candidate solutions for both impactor and target that sat-

isfy the initial conditions of the problem. These solutions are then matched in a parameter plane of contact interface stress σ^* vs. contact interface velocity v^*. In view of the issues involving dynamical nonuniqueness mentioned previously, numerous such matches may be possible, all of which are either dissipative or else energy conserving. In a space-time diagram (the (x,t)-plane) the solution to the Riemann problem partitions itself into sectors, and in each sector the dynamical fields may either involve constant strain and velocity (a C-sector), or else a continuous compressive wave (a W-sector). The interface between sectors may be continuous, a conventional shock (S) or else a phase boundary (P). We first considered the impact between a target material that admitted phase transformations, and an impactor with a more conventional (albeit nonlinear) response (Pence 1990a, 1992a). We considered a range of impact velocities $\bar{v}$ that allow a variety of complicated wave patterns involving phase transitions in the target material. In this regard we established the existence of various threshold values of impact velocity $\bar{v}_1 < \bar{v}_2 < \bar{v}_5 < \bar{v}_N$ (adopting the notation of Pence 1992a) such that:

(i) For $0 < \bar{v} < \bar{v}_1$ there is a single solution with wave pattern (in the target) of type C-W-C, and, in view of the absence of shocks and phase transitions, this solution is energy conserving.

(ii) For $\bar{v}_1 < \bar{v} < \bar{v}_2$ there are exactly two solutions, one of type C-W-C and the other of type C-W-C-P (starting from the x-axis and proceeding to the t-axis). Here the latter differs from the former only by the presence of a stationary (and hence non-dissipative) phase boundary on the interface of contact (the t-axis in a Lagrangian frame). Both such solutions are energy conserving.

(iii) For $\bar{v}_2 < \bar{v} < \bar{v}_5$ there is a one-parameter *family* of solutions, each of which is either of type C-W-C-P or C-W-P or C-W-P-C or C-W-C-P-C or C-P-C. The bounding solutions of this family may be energy conserving, while the interior solutions are strictly dissipative. Thus, at most 2 of these solutions are energy conserving. In addition, there may or may not exist a single (energy conserving) solution of type C-W-C.

(iv) For $\bar{v}_5 < \bar{v} < \bar{v}_N$ there is exactly one solution with wave pattern of type C-P-C and this solution dissipates energy.

The impact velocity regime $\bar{v} > \bar{v}_N$ involves complicated (single-phase) wave patterns in the impactor, but no additional phase transitions in the target, and so the analysis was not pursued into this regime. The far more significant velocity regime (iii) of $\bar{v}_2 < \bar{v} < \bar{v}_5$ was, however, analyzed at great detail. It was found that the (σ^*,v^*)-plane provides a natural basis for investigating the effect of different solution selection criteria. In Pence (1992a) the maximal dissipation selection criteria was examined by determining the qualitative features of the level curves of dissipation in the (σ^*,v^*)-plane. It was shown that there exists another threshold velocity $\bar{v}_6$ obeying $\bar{v}_2 < \bar{v}_6 < \bar{v}_5$ such that:

(iiia) For $\bar{v}_2 < \bar{v} < \bar{v}_6$ maximally dissipative solutions are restricted to the interior of the one-parameter family and necessarily have wave pattern C-W-C-P-C. Such solutions may or may not be unique. However, certain special forms of the stress-response function for the target material will ensure uniqueness.

(iiib) For $\bar{v}_6 < \bar{v} < \bar{v}_5$ there is exactly one maximally dissipative solution, and it is one of the bounding solutions of the one-parameter family. The resulting wave pattern is of type C-W-P-C.

The particular special forms of the stress-response function which were shown to ensure uniqueness in (iiia) correspond to convexity and spacing conditions on the global well structure of the elastic energy density.

The extent to which particular dynamical states can be generated (or controlled) can be formulated

as an inverse Riemann problem (Pence 1990b). Specifically, one seeks to determine whether a given dynamical wave pattern in the target involving a moving phase boundary, can be generated by judicious choice of impactor material and impact velocity. Such information could be useful for example in designing explosive cladding processing/synthesis operations for adaptive composites involving shape memory alloy surface structures. The particular dynamical states that are so attainable were determined in terms of the (σ^*,v^*)-plane, and an algorithm was given for the determination of the associated impactor material and impact velocity. Although the particular study was limited to dynamical phase transformations governed by the maximum dissipation criterion, the same methodology could easily be applied to other selection criteria such as those involving a kinetic relation.

Impact between two materials, each of which is capable of phase transformations, is the focus of Pence (1992b); the (σ^*,v^*)-plane remains a useful organizing principle. In this case, if phase transformations take place simultaneously in the impactor and target, then certain ranges of impact velocity lead to a two-parameter family of dynamical state solutions prior to the invocation of a dynamical selection criterion. Properties of the level curves of dissipation were obtained for this problem, although their characterization is neither as simple nor as complete as that for the problem involving only a single phase transforming material (Pence 1992a). It was, however, established that the maximally dissipative selection criterion may lead to a symmetry breaking bifurcation phenomena similar to that which occurs in Euler column buckling. Recall in Euler buckling that pairs of "equally likely" asymmetric buckled states (i.e. right buckled & left buckled) bifurcate from the symmetric (unbuckled) state at a particular value of load. In an analogous fashion, the symmetric impact of two identical phase transforming materials may give symmetric wave patterns for low values of impact velocity, while at some threshold value of impact velocity, the solutions may bifurcate into a pair of "equally likely" asymmetric dynamical wave patterns in the two materials.

This section summarizes results that are reported in the following four publications:

Pence, T.J. (1990a). Phase transitions and maximally dissipative dynamic solutions in the Riemann problem for impact. Transactions of the Eighth Army Conference on Applied Mathematics and Computing, F. Dressel ed. 817-832.

Pence, T.J. (1990b). An inverse Riemann problem for impact involving phase transitions. Transactions of the Eighth Army Conference on Applied Mathematics and Computing, F. Dressel ed. 833-846.

Pence, T.J. (1992a). On the mechanical dissipation of solutions to the Riemann problem for impact involving a two-phase elastic material. *Arch. Ration. Mech. & Anal.* **117**, 1-52.

Pence, T.J. (1992b). The dissipation topography associated with solutions to a Riemann problem involving elastic materials undergoing phase transitions. to appear in *Shock Induced Transitions and Phase Structures in General Media,* R. Fosdick, E. Dunn & M. Slemrod eds., IMA volume series, Springer-Verlag.

6. Thermal Effects Upon Phase Boundary Mobility

In order to predict the temperature change associated with a mechanically activated phase transition we have extended the framework of Pence (1991a,b) in order to encompass the role of thermal activation or inhibition. The utility of considering such extended theories is demonstrated by the work of Slemrod (1983, 1984) and Hattori (1986a,b). To this end, we have developed a model that is thermodynamically separable in that it can be embedded into a fully thermodynamically consistent theory in which the internal energy ε as a function of both shear strain γ and specific entropy η additively decouples into a separate functions of each (this idea dates back to Courant and

Friedrichs 1948). Mechanically activated motion is isothermal within such materials. In order to extend our modeling capability to account for thermomechanical coupling, we have extended the thermodynamical framework to a wider class of materials which encompass the material treated in Pence (1991a,b) and Lin and Pence (1991, 1992a,b) as a special case. The research described in this section is the basis of graduate student Ralph Worthington's Ph.D. research.

We consider a material in which each phase, say indexed by i, has internal energy ε, stress τ and entropy η as functions of shear strain γ and temperature θ given by

$$\varepsilon_i = \frac{1}{2}\mu_i[\gamma - a_i]^2 + \bar{b}_i\theta + \hat{b}_i\,, \qquad \tau_i = \mu_i[\gamma - a_i] - \bar{k}_i\theta\,, \qquad \eta_i = \bar{b}_i\ln\theta + \bar{k}_i\gamma + \hat{k}_i\,,$$

where μ_i, a_i, $\bar{b}_i$, $\hat{b}_i$, $\bar{k}_i$ and $\hat{k}_i$ are constitutive parameters which determine the spacing and curvature of the individual phase energy wells, as well as the thermomechanical coupling. In particular, $\bar{k}_i$ is a thermomechanical coupling parameter since any phase for which $\bar{k}_i = 0$ is governed by the purely mechanical theory treated in Pence (1991a,b) and Lin and Pence (1991, 1992a,b). The equations of compatibility, force balance and energy balance give a system of partial differential equations similar to that obtained by Niezgodka and Sprekels (1988) in their studies of phase transformations, albeit of different form since the energy potential is not of Landau-Devonshire type. More significantly it does not allow field resident viscous dissipation, and there is also the assumption that heat conduction is negligible compared to the other process variables, thus having the advantage of yielding a model with distinct phase interfaces. This model is in fact qualitatively similar to the general class of models described by James (1983), in addition it also incorporates the coupling parameter $\bar{k}_i$ *which enables us to investigate how the thermomechanical* theory unfolds from a purely mechanical theory whose behavior is well known.

We have examined the encounter of a pulse with a phase boundary in this thermomechanical framework for the case in which $\bar{k}_1 = 0$, and $\bar{k}_2 > 0$ corresponding to a thermally active second phase. Consider for the moment a thermally inactive second phase (both $\bar{k}_1 = 0$, and $\bar{k}_2 = 0$). Then the pulse-phase boundary encounter problem requires the consideration of two regions in a space-time plane: a region corresponding to the transmitted pulse and a region corresponding to an interaction zone (bounded by the incoming pulse, the phase boundary, and the reflected pulse). By means of characteristic theory, four fundamental (linear) relations between the strain and velocity in these two regions can then be obtained (Pence 1991a). When these equations are solved in terms of the strain magnitude of the incoming pulse $\Delta\gamma$, the phase boundary speed is not determined and hence is obtained by an external mobility criterion of the type discussed in Truskinovsky (1990), Pence (1991a,b), and Abeyaratne and Knowles (1991). Let us now return to the case of a thermally active second phase $\bar{k}_2 > 0$ and assume that the pulse is incident through the thermally inactive first phase and that the phase boundary is driven into the thermally active second phase. One then finds that the interaction zone now splits in two along a (new) characteristic associated with the temperature variable. There are then nine fundamental field quantities (strain, velocity and temperature in each of the three regions). By means of characteristic theory, we have obtained nine equations relating these field quantities to the disturbance magnitude $\Delta\gamma$ and the phase boundary speed $\dot{s}$. These equations are not linear in the temperatures. We have been able to reduce these equations to a single highly nonlinear algebraic equation for the temperature θ_T in the transmitted wave adjacent to the

phase boundary:

$$\sqrt{\bar{b}_2}\,[\sqrt{\mu_1} - \dot{s}\sqrt{\rho}]\, f(\theta_T) - \frac{\bar{b}_2}{\bar{k}_2}[\dot{s}\sqrt{\rho\mu_1} - \mu_2]\ln\theta_T + \bar{k}_2\theta_T =$$

$$\sqrt{\bar{b}_2}\,[\sqrt{\mu_1} - \dot{s}\sqrt{\rho}]\, f(\hat{\theta}) - \frac{\bar{b}_2}{\bar{k}_2}[\dot{s}\sqrt{\rho\mu_1} - \mu_2]\ln(\hat{\theta}) + \bar{k}_2\hat{\theta} + 2\Delta\gamma[\dot{s}\sqrt{\rho\mu_1} - \mu_1] + \dot{s}\sqrt{\rho\mu_1}\,[\gamma_1 - \gamma_2],$$

where $f(\theta) = 2\Phi(\theta) + \Phi(0)\ln\left[\frac{\Phi(\theta) - \Phi(0)}{\Phi(\theta) + \Phi(0)}\right]$ and $\Phi(\theta) = \left[\frac{\mu_2\bar{b}_2}{\bar{k}_2^2} + \theta\right]^{\frac{1}{2}}$. Here ρ is the material mass density and $\hat{\theta}$ is the ambient temperature in the material before the encounter. The results given in Pence (1991a) with $c_i = \sqrt{\mu_i/\rho}$ must be retrievable in the limit $\bar{k}_2 \to 0$. Preliminary indications are that this is indeed the case, for example we find for small $\bar{k}_2$ that the above equation yields

$$\theta_T = \hat{\theta}\exp\left(\frac{\bar{k}_2\Gamma}{\bar{b}_2\xi}\right)\left[1 + \frac{\bar{k}_2^3\Gamma\hat{\theta}}{\bar{b}_2\xi^2}\left[\frac{\sqrt{\mu_1} - \dot{s}\sqrt{\rho}}{2\sqrt{\mu_2}} + 1\right] + o(\bar{k}_2^3)\right],$$

where $\Gamma = (\dot{s}\sqrt{\mu_1\rho} - \mu_1)\,2\Delta\gamma + \dot{s}\sqrt{\mu_1\rho}\,(\gamma_1 - \gamma_2)$ and $\xi = \rho\left(\sqrt{\frac{\mu_1}{\rho}} + \sqrt{\frac{\mu_2}{\rho}}\right)\left(\sqrt{\frac{\mu_2}{\rho}} - \dot{s}\right)$. We then find that a long calculation gives, for example, that the velocity in the transmitted wave is $v_T = v_T^{mech} + v_T^{\bar{k}_2}$ where v_T^{mech} is the velocity for the problem in which $\bar{k}_2 = 0$:

$$v_T^{mech} = \sqrt{\frac{\mu_1}{\rho}}\sqrt{\frac{\mu_2}{\rho}}\left[\frac{\left(\dot{s} - \sqrt{\frac{\mu_1}{\rho}}\right)2\Delta\gamma + \dot{s}\,(\gamma_1 - \gamma_2)}{\left(\sqrt{\frac{\mu_1}{\rho}} + \sqrt{\frac{\mu_2}{\rho}}\right)\left(\sqrt{\frac{\mu_2}{\rho}} - \dot{s}\right)}\right]$$

and $v_T^{\bar{k}_2}$, which is order $\bar{k}_2^2$, is given by

$$v_T^{\bar{k}_2} = \left(\frac{\bar{k}_2^2\Gamma\hat{\theta}}{\bar{b}_2\xi}\right)\left[\frac{1}{\xi}\sqrt{\frac{\mu_2}{\rho}}\left(\frac{\sqrt{\mu_1} - \dot{s}\sqrt{\rho}}{2\sqrt{\mu_2}} + 1\right) + \frac{1}{2\sqrt{\rho\mu_2}}\right] + o(\bar{k}_2^2).$$

As expected, the leading order term in the above result coincides with that given in Pence (1991a) once allowance is made for the pulse being incident through phase-1. We anticipate that all of the nine field quantities will similarly yield to a formal asymptotic series where the leading order term retrieves the results presented in Pence (1991a). These results will enable us to precisely gauge the extent to which purely mechanical theories of phase boundary motion can be unfolded into fully thermomechanical theories. An assessment of the qualitative robustness of this unfolding process then demands an inquiry as to whether or not completely new solutions appear during this unfold-

ing process. These questions, and related issues, are currently under active investigation.

The results presented in this section summarize the results of collaboration with Ph.D. graduate student Ralph Worthington. Ultimately, in addition to Mr. Worthington's Ph.D. thesis, we estimate that at least two archival papers will result, which at present we are targeting for the *SIAM J. of Applied Math*, and for *Continuum Mechanics and Thermodynamics*.

7. Thermal and Mechanical Evolution of Phase Variants

All of the results discussed thus far are devoted to the modeling of phase boundary motion at the micromechanical level. The leap to a macromodel requires the consolidation of the various effects, describable individually at the microlevel, into a constitutive theory which accounts for their cumulative effect at a level of detail appropriate for the model's application area. To this end, we have been collaborating with an industrial counterpart (Y. Ivshin of Johnson Controls) in order to bridge this gap in a fashion that incorporates the ever increasing knowledge base obtained from the type of detailed micromechanical modeling discussed thus far.

Both the shape memory effect and pseudo-elastic effect are examples of hysteretic response in the austenite A to martensite M transition. For a given shape memory alloy active element, this response can be characterized experimentally for complete transition paths (pure A to pure M and vice-versa) by various standard techniques (calorimetry, x-ray diffraction, resistivity, load-cell). The pure A to pure M transition will thus give rise to a response surface in the relevant parameter space of design variables (e.g. temperature, stress, displacement), and the pure M to pure A transition will give rise to a similar, but distinct, surface. Together these two surfaces form an envelope which bounds the system response for all possible transition paths. Its significance derives from the fact that, within a precise sensing and control application, an active element will invariably undergo partial transitions before arresting itself and reversing its response, thus generating a complicated trajectory within the bounding envelopes. Algorithms for predicting these trajectories from the knowledge of the complete transition behavior are of fundamental concern, as it is impractical to experimentally compile a complete data base description of all arrested/reversed transition behavior for a given shape memory alloy active element (i.e. to experimentally determine the trajectories of all space filling curves within the bounding envelopes). Analytical characterization of these trajectories is also, of course, central to a continuum based constitutive description of such material behavior.

Various algorithms have been proposed for the modeling of shape memory material constitutive response (e.g. Muller 1979; Muller and Wilmanski 1980; Liang and Rogers 1990; Zhang, Rogers and Liang 1991), but in general the precise characterization of arrested/reversed transition behavior in terms of complete transition behavior has not been the central concern. When it has been, issues related to proper qualitative algorithmic behavior have not been completely addressed, thus possibly leading to models which, for example, allow certain response trajectories involving arrested/reversed transitions to exit the bounding envelopes. In Ivshin and Pence (1992a) we have given examples of how seemingly reasonable hysteresis algorithms can give rise to such cases of envelope piercing. To address such issues we have developed a model for rate independent hysteresis, governed by differential equations of Duhem Madelung type (Visintin 1988), which is particularly applicable to temperature induced phase transitions which occur in shape memory alloys. Other physical systems exhibiting hysteresis have been modeled within a similar framework, notably ferromagnetically soft materials (Coleman and Hodgdon 1986, 1987). In its simplest form, as examined

in Ivshin and Pence (1992a), the model treats the evolution of a relative phase fraction variable (e.g. austenite fraction) due to a prescribed temperature history. Micromechanical considerations, notably that partial transitions proceed at a pace that is proportional to the phase fraction of the parent phase (the phase A or M that is being depleted), are ensured by the form of the governing relations. This leads to a correspondence between experimentally determinable transition envelopes for complete transitions, and thermodynamic phase fraction transition rate functions.

We believe that this model extends previous modeling efforts in three key respects. First, connection to experiment follows directly from the dependence upon a hysteresis envelope for complete transitions, and envelope restrictions have been obtained which ensure proper monotonicity, containment and orientation requirements upon partial transition paths. Second, we have identified underlying mathematical structures which rigorously characterize the hysteresis response. These structures include: attracting limit cycles for finite cycling, and attracting limit points for infinitesimal cycling (drift). These structures are anticipated to allow for the rational analysis of seemingly chaotic response trajectories. Third, although many models for the shape memory affect are capable of capturing isothermal stress-strain hysteresis, the generalization to adiabatic and convective heating has not received appropriate attention, especially since the latter is critical for device simulation. We have successfully simulated isothermal, adiabatic, and convective stress-strain behavior in a simple prototype model. In particular, the general convective stress-strain behavior successfully approaches adiabatic behavior for low convectivity/fast loading, and successfully approaches isothermal behavior for high convectivity/slow loading. These results also allow for simulation of multiple heating/cooling mechanisms acting simultaneously (e.g. resistive heating/ convective cooling). Since the prepublication manuscript which details this successful simulation of convectivity behavior is still in preparation, we present a short summary diagram in Figure 1.

In Ivshin and Pence (1992b), we have implemented the minimum amount of extensions to Ivshin and Pence (1992a) that allow for a micromechanically consistent modeling of the two-way memory effect. This requires the consideration of two martensite variants: M+ and M-, with different transformation strains. The underlying phase transitions between the three phase/variant species A, M+ and M- are then triggered both mechanically and thermally on the basis of changes in material stability as governed by an enlarged set Duhem-Madelung differential equations. In Ivshin and Pence (1992b), we considered a simple symmetric three layer composite system, each layer of which is composed of the same shape memory material. Unequal initial lengths prior to bonding gives rise to a piecewise constant residual stress distribution as one proceeds through the layers. For this simple system, the two way memory effect manifests itself in terms of length changes which occur upon raising and lowering the system temperature. This behavior stems from the ability of the model to account for asymmetry in transitions between the three phase/variant species owing to the nonuniform nature of the residual stress distribution. A complete description of this system required the consideration of eleven variables: the common system temperature; the stress, strain and phase fractions of A, M+ and M- in the (symmetric) outer layers; and the stress, strain and phase fractions of A, M+ and M- in the middle layer. Ten differential equations relating these variables follow naturally from the model, which, when augmented with a prescribed applied temperature T(t), gives rise to a well defined mathematical system. Numerical simulations for a periodic T(t) exhibited cyclic length changes to the overall system, which rapidly collapsed onto a limit cycle when graphed against temperature. The other system variables also exhibited a rapid collapse onto their associated limit cycles.

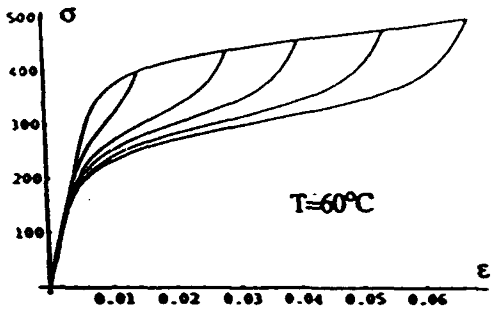

Figure 1a: Simulated uniaxial stress-strain behavior at 60°C, which is greater than A_F, exhibiting a stress plateau due to transformation strain from martensite nucleation. Unloading proceeds along different curves, with complete reversion to austenite near 175 MPa. The response is independent of loading rate.

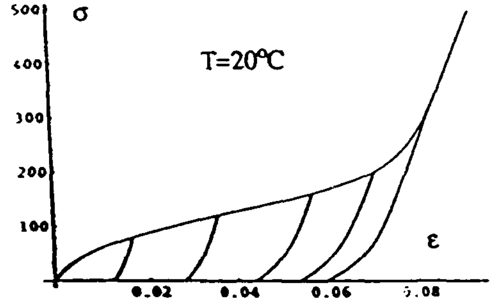

Figure 1b: Simulated stress-strain behavior at 20°C, which is less than A_F, exhibiting essentially complete transformation to martensite near 300 MPa. Unloading from different levels results in different amounts of retained martensite.

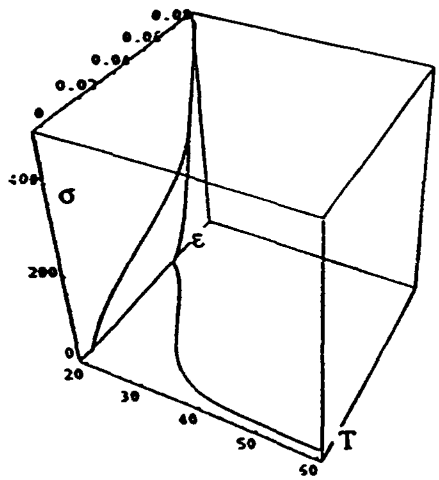

Figure 1c: Increasing the temperature after the outermost simulation of Fig. 1b results in the reversion of retained martensite to austenite, and hence disappearance of permanent deformation due to transformation strain.

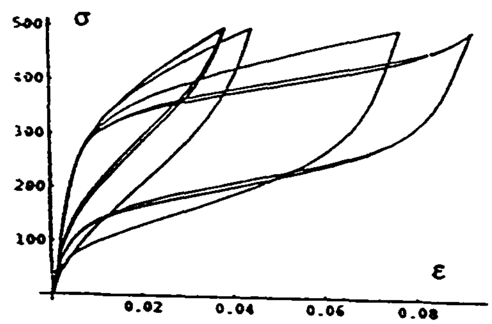

Figure 1d: Isothermal response at 50°C (far right) and adiabatic response beginning at 50°C (far left) are both rate independent. The four intermediate curves give response with convective heat transfer (quiet air) to an ambient 50°C for loadings which conclude in 4, 40, 400 and 4000 sec respectively (left to right). The 4s loading is nearly adiabatic, whereas the 4000s loading is nearly isothermal.

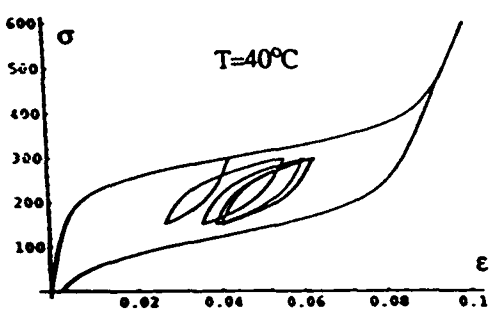

Figure 1e. Isothermal response at 40°C for loading/unloading to 600 MPa, and for loading to 300 MPa followed by cycling five times to and from 150MPa. Convergence to a limit cycle is evident.

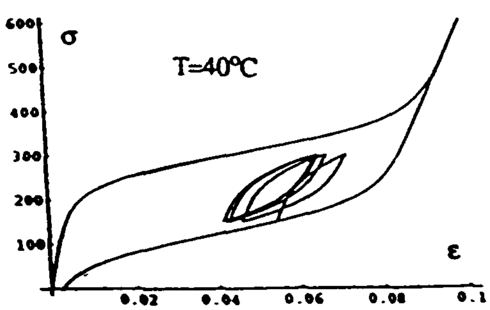

Figure 1f. Isothermal response at 40°C for loading/unloading to 600 MPa (as in Fig. 1e), and for arrested unloading at 150MPa followed by cycling five times to and from 300MPa. Convergence to the limit cycle of Fig. 1e again takes place, but now from the other side.

This section summarizes results that are reported in the following two publications:

Ivshin, Y. and T. J. Pence (1992a). A constitutive model for hysteretic phase transition behavior. (accepted for publication in *Int. J. Eng. Sci.*).

Ivshin, Y. and T. J. Pence(1992b). A simple mathematical model of two-way memory effect. (to appear in *Proceedings of the 1992 Int. Conf. on Martensitic Transformations* (ICOMAT-92)).

In addition, another manuscript is substantially complete which we intend to submit in early Spring 1993 to either the *Int. J. Eng. Sci*, or to *J. Intelligent Materials and Structures.*

8. Laminated Structures and Thin Films

Within the setting of finite deformation elasticity, the ability of a particular material model to admit the abrupt nucleation of phase boundaries and other microstructural features responsible for qualitatively different macroscopic response is dependent upon the constitutive energy function of the material. In this regard, material instabilities at the macrolevel originate as structural instabilities at the microlevel. The associated bifurcation phenomena, whose behavior may be well understood in monolithic systems, can drastically change its character in composite systems. In highly deformable thermoelastically "passive" materials (they do no not admit phase changes), our previous work has shown how the buckling of layered structures leads to configuration modification, and buckling load reordering, for those buckled states that are natural extensions of previously known solutions corresponding to an isolated layer. For the isolated layer it had been shown (Sawyers and Rivlin 1974) that there are two families of buckled solutions: a conventional antisymmetric *flexural* buckling mode (reminiscent of classical Euler buckling), and a symmetric *barreling* buckling mode. The buckling loads associated with these different buckled solutions are ordered:

$$T_1^F < T_2^F < \ldots < T_m^F < T_{m+1}^F < \ldots \rightarrow T_\infty \leftarrow \ldots < T_{m+1}^B < T_m^B < \ldots < T_2^B < T_1^B,$$

where T_m^F and T_m^B are the buckling loads associated with the m^{th} flexure and barreling mode respectively, and T_∞ is associated with a wrinkling instability. In contrast, for a three-ply sandwich construction we find certain instances in which T_∞ is the minimum buckling load so that wrinkling, and not mode-1 flexure, is the critical instability (Pence and Song 1991).

These buckling results have immediate consequences for the optimal design of laminated structures with respect to buckling resistance. This was demonstrated in Song and Pence (1992) for the following simple pilot problem: *Construct a symmetric three-ply layered composite of given dimensions from two neo-Hookean materials, A and B, whose combined amount is equal to the total volume. Thus either A or B may be used for the central ply. The optimal construction is that which gives the largest critical thrust.* We found that the optimal design was dependent on the aspect ratio of the construction. If the structure is short in the direction of thrust, then the optimal construction was one in which the stiffer material was used in the central ply. However, if the structure is long in the direction of thrust, then the other construction is optimal. Thus there is a transition aspect ratio in this simple optimal design problem. Numerical routines were developed to determine this aspect ratio. In addition, by introducing composite construction characterization parameters β and α, where β is the ratio of the ply stiffnesses and α is the volume fraction of the central ply, we successfully implemented a (β,α)-parameter plane characterization of the optimal design criteria.

Recently, we have established the existence of completely new buckled states that have no isolated

layer counterpart. For example, it appears for an N-layered structure that there exists N-1 new families of buckled solutions, in addition to the 2 previously known families corresponding to flexure and barreling (we have verified this for N=2,3 while for N>3 this is, at present, our working hypothesis). We are developing appropriate numerical algorithms for unraveling the complicated bifurcation behavior and clarifying the properties of these new solutions. In particular, asymptotic analysis has enabled us to determine precise load levels which lead to wrinkling phenomena. In addition these new deformations will be neither pure flexure nor pure barreling, and we are developing methods for characterizing this mode-mixity (the importance stems from the fact that the different mode types may be prone to different delamination risk). Although the analysis is at present limited to passive nonlinear elastic materials, we are here building toward the goal of analyzing multi-layers that involve both passive and thermoelastically active layers, since the presence of the latter offers the possibility for active mechanical tuning of an overall smart structure. The results given in Ivshin and Pence (1992b) indicate that this can be accomplished in thermoelastically active multi-ply structures for simple uniaxial deformations, whereas here we are concerned with more complicated buckling phenomena as well as more general out of plane displacement modes.

The results of our study of laminated structures are applicable to thin films in an appropriate limiting case. As a practical matter, thermoelastic phase transformation behavior is understood far better for materials in bulk form than it is for thin film structures. Here the influences low dimensional constraint; specialized fabrication induced variant structures (and their associated residual stress fields); and the competition between translaminar and intralaminar heat flow all serve to complicate the structural response. We have recently begun to investigate the possibilities for modeling these phenomena (an avenue of study which was completely unforeseen in 1989 when the present study was proposed). We hope to pursue this investigation via collaboration with materials science colleague David S. Grummon. Recently, Prof. Grummon has successfully fabricated NiTi amorphous thin films, and subsequently has developed annealing methods to achieve crystallographic structure which enable thermoelastic austenite/martensite transformations that underlie shape-memory and superelasticity effects. This is the rationale for our joint proposal to ARO, which was mentioned earlier in this report.

This section summarizes results that are reported in the following publication:

Song, J. and T. J. Pence (1992), On the design of three-ply nonlinearly elastic composite plates with optimal buckling performance, *Structural Optimization* 5 (1992) 45-54.

In addition, another manuscript is substantially complete which we intend to submit in late winter 1993 to *Int. J. Solids and Structures.*

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10. Academic History of Principal Investigator T.J. Pence

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Education

Ph.D.	Applied Mechanics	California Institute of Technology	1983
M.S.	Applied Mechanics	California Institute of Technology	1980
B.S.	Engineering Mechanics	Michigan State University	1979

Professional Experience

1990 - Associate Professor, Department of Materials Science and Mechanics, Michigan State University

1986 -1990 Assistant Professor, Department of Materials Science and Mechanics, (previously the Department of Metallurgy, Mechanics and Materials Science) Michigan State University

1983 -1986 Research Fellow, Mathematics Research Center, and Van Vleck Assistant Professor, Department of Mathematics, University of Wisconsin-Madison

1982 -1983 Postdoctoral Scholar and Lecturer, Department of Applied Mechanics, California Institute of Technology

1980 -1982 Graduate Teaching Assistant, Department of Applied Mechanics, California Institute of Technology

Visiting Positions

9/90 - 10/90 Visiting Scholar, Institute for Mathematics and its Applications, University of Minnesota - Minneapolis

9/84 - 6/85 N.A.T.O. Postdoctoral Fellow, Laboratoire d'Analyse Numerique, Universite de Paris - Orsay

6/83 - 7/83 Summer Intern, Offshore Systems Division, Exxon Production Research Co., Houston, TX

6/79 - 9/79 Summer Intern, Mechanical Properties Group, Argonne National Laboratory, Argonne, IL

6/78 - 9/78 Summer Intern, Logic Device and Systems Division, Cutler Hammer (now a division of Eaton Corp.), Fenton, MI

Memberships

American Academy of Mechanics, American Society for Engineering Education (ASEE), American Society of Mechanical Engineers (ASME), Materials Research Society (MRS), Society for Natural Philosophy, Society of Engineering Science (SES), Society of Industrial and Applied Mathematics (SIAM).